THEORETICAL-EXPERIMENTAL STUDY OF COMPLEX HEAT TRANSFER IN HIGHLY POROUS COMPOSITES

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UDC 536.21:536.33

A probabilistic approach is proposed for modeling the physical characteristics of high-porosity randomly nonuniform media. The results of mathematical modeling of complex heat transfer are compared with experimental data.

High-porosity composites serve as basic elements of different types of systems designed to provide protection from the effects of heat. A tightening of the requirements established for these systems for the service conditions encountered by certain types of equipment is making it more important to develop materials with prescribed properties that will have the optimum service characteristics under the given conditions.

One distinctive feature of highly porous composites is the fact that three heat-transfer processes can take place in them: conduction, convection, and radiation. The intensity of these processes is determined by the thermal loading conditions on the one hand and, on the other hand, by the material's physical characteristics (thermophysical, optical, hydraulic). These characteristics in turn depend on the structure of the material. Thus, the material's structure and the conditions of its use are the main factors which determine heat transfer in highly porous composites [1, 2].

The problem of developing thermal insulating material with prescribed properties cannot be solved without developing a mathematical model that links the structure of the material with its service conditions and its thermal state during service.

Mathematical Model of Radiative-Convective Heat Transfer in a Layer of a High-Porosity Composite. The main contribution to heat transfer in a wide range of high-porosity composites (HPC) is made by radiative-convective heat transfer (RCT) [3].

The mathematical model of RCT in a plane layer of an HPC bounded by opaque walls and having the length l includes the energy equation with the associated initial and boundary conditions and the equation (with corresponding boundary conditions) that describes radiative transfer in the diffusion approximation. The diffusion approximation is widely used to analyze RCT, combining good accuracy with relatively short computing times [4]:

$$c(T(x,\tau))\frac{\partial T(x,\tau)}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda(T(x.\tau),P(\tau))\frac{\partial T(x,\tau)}{\partial x}\right)-$$

$$-\frac{\partial}{\partial x}(Q_p(x,\tau)),\quad 0<x<l,\quad 0<\tau\leqslant\tau_m; \tag{1}$$

$$T(x,0)=T_0(x); \tag{2}$$

$$T(0,\tau)=\varphi_1(\tau);\quad T(l,\tau)=\varphi_2(\tau);\quad P(\tau)=\varphi_3(\tau); \tag{3}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{\alpha(x)}\frac{\partial F(x,\tau)}{\partial x}\right)-3b(x)F(x,\tau)=$$

$$=4\pi n^2\frac{\partial I_p(x,\tau)}{\partial T}\frac{\partial T(x,\tau)}{\partial x}; \tag{4}$$

$$Q_p(x,\tau)=\int_0^\infty F(x,\tau)\,d\nu; \tag{5}$$

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 59, No. 4, pp. 554-561, October, 1990. Original article submitted December 5, 1988.

0022-0841/90/5904-1238$12.50

$$\frac{1}{\alpha(0)}\frac{\partial F(0,\tau)}{\partial x}=F(0,\tau)\frac{4-2a_{11}}{a_{10}}; \quad (6)$$

$$\frac{1}{\alpha(l)}\frac{\partial F(b,\tau)}{\partial x}=-F(l,\tau)\frac{4-a_{21}}{a_{20}}. \quad (7)$$

In Eqs. (4-7), b = α + β(1 − a); a is the mean cosine of the scattering angle; $a_{i0}=2\int_0^1\varepsilon_i(\mu)\mu d\mu$; $a_{i1}=3\int_0^1\varepsilon_i(\mu)\mu^2/d\mu$ $(i=1,2)$ are the instantaneous values of the optical characteristics of the boundaries [4].

The problem is concretized by assigning initial and boundary conditions for the energy and radiative transfer equations along with the values of the physical characteristics: volume heat capacity c(T) and thermal conductivity λ(T, P); spectral absorption coefficient α; spectral scattering coefficient β; refractive index n; scattering function γ(μ); optical characteristics of the boundaries ε_1, ε_2. The values of the optical and thermophysical characteristics depend on the structure of the material. Thus, we will take a closer look at the features of this structure.

Structure of Fiber Composites. The study of the structure of HPC's has shown that they generally have several distinctive features that must be taken into account (Table 1) when developing mathematical models.

1. The composition of the HPC may include fibers made of different materials [5, 6] and thus having different thermophysical and optical characteristics. As a result, the structure-forming elements (fibers) are distributed with respect to the physical characteristics. Also, each composition is characterized by a certain ratio of the volume fractions of the components which enter into it.

2. The fibers are also distributed according to size (length and diameter). The histograms describing this distribution and the main probability characteristics – the mathematical expectation and the dispersion – depend on features of the process used to make the HPC's and the raw materials used [1, 5, 7].

3. The fibers are additionally distributed according to orientation. The features of this distribution account for the anisotropy of the properties of HPC's [5].

4. A contact zone is formed at places where the fibers touch one another, the formation of this zone leading to additional thermal resistance [6, 8]. The degree of thermal resistance due to the zone depends on the material of the binder and the dimensions of the zone, which may in the general case be random variables.

5. The structure of HPC's may contain macroscopic defects in the form of pores, cracks, granules, fibers that have stuck together, globular particles, etc.

6. Different compositions have different apparent densities. Scatter of the apparent density and distribution of this characteristic over the volume of the material are possible even for a given grade of HPC.

The random character of the parameters of the structure-forming elements makes it necessary to regard HPC's as randomly nonuniform media and to use probability-based methods to perform the modeling.

TABLE 1. Features of the Structure of Fiber Composites

Material characteristic	Material				Characteristics of the fibers			
	Li-900	FRCI-20	FRCI-80	AETB	Thermal conductivity λ, W/(m·K)		Mean diameter d_f, μm	Length ℓ_f, μm
Density, kg/m³	140—320	190—320	190—320	320				
Anisotropy	0,9	0,5	0,5	0,5	T = 500 K	T = 1500 K		
Volume fraction of the SiO_2	1	0,8	0,2	0,3	1,9	2,6	1,2—4	150—1000
Alumoborosilicate	0	0,2	0,8	0,4	4,7	3,6	11	3175
Al_2O_3	0	0	0	0,3	17,7	7,37	3	1270

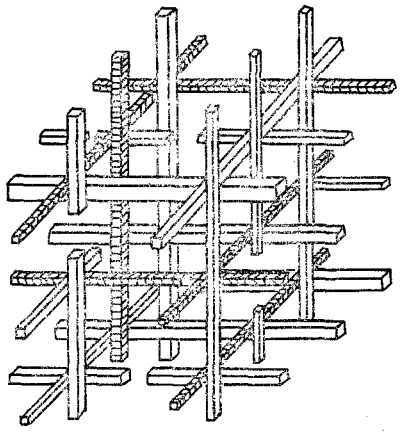

Fig. 1. Model structure.

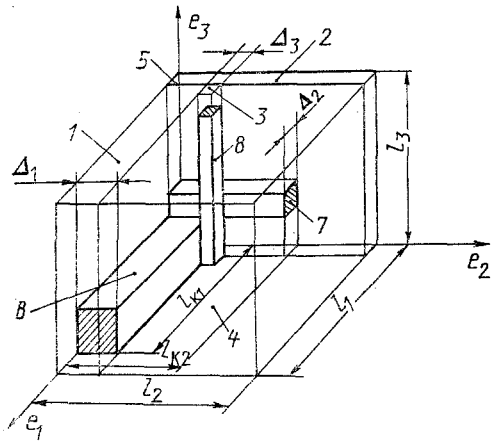

Fig. 2. Elementary volume.

The most important step in the development of mathematical models of physical characteristics is the choice of a model structure.

Model Structure and Determination of Its Parameters. The model of the structure of a fiber composite is the foundation on which the model of physical characteristics is built. A study of the literature [5, 6, 9, 10] showed that the structural models developed earlier were deterministic and were oriented toward allowing for some structural feature while ignoring the effect of different features on each other and the stochastic character of the parameters of the structure-forming elements. This approach seriously limits the range of application of the model and makes it necessary to develop a model structure that satisfies two conflicting requirements – approximate the original structure as closely as possible and be amenable to mathematical modeling. As such a structure, we propose a regular, orthogonal, randomly nonuniform system of fibers (Fig. 1) in which the distribution of the fibers with regard to size and thermophysical characteristics is the same as in the original material. The main assumptions made are that the system of fibers is orthogonal and regular. We can use a regular structure because the fibers are positioned relatively uniformly in space.

In order to substantiate the second assumption, we conducted special studies in which we verified the possibility of changing over from a nonorthogonal structure to an orthogonal structure while keeping the same values for the main characteristics: apparent density and thermal conductivity along the principal axes [2, 3]. Such a transition is possible if the following conditions are satisfied:

$$\rho_c = M[\rho_{e.v}(l_1,\ l_2,\ l_3,\ \xi)], \tag{8}$$

$$l_1/l_3 = M[(\bar{l}_1/\bar{l}_3)^2], \tag{9}$$

$$l_2/l_3 = M[(\bar{l}_2/\bar{l}_3)^2], \tag{10}$$

where ρ_c and $\rho_{e.v}$ are the apparent densities of the HPC and an elementary volume isolated within the model structure; ℓ_i (i = 1, 3) are the dimensions of the elementary volume along the axes, these dimensions being equal to the dimensions of the model structure; ℓ_i(i = 1, 3) are projections of a fiber on the principal axes.

These conditions make it possible to determine the characteristic dimensions of the model structure. The selection of a model structure and determination of its parameters make it possible to proceed to the next step – calculate the thermophysical characteristics of composites.

Mathematical Modeling of the Physical Characteristics of Fiber Composites. We will isolate an elementary volume in the model structure (Fig. 2), this volume reflecting the features of the structure and being such that it is possible to calculate the value of a certain physical characteristic (such as thermal conductivity) for it. The value of the physical characteristic ϕ_i of the elementary volume depends on the random vector of the parameters of the volume

$$\xi = (\Delta_j,\ l_{fj},\ l_{sj},\ \rho_j,\ h_{sj},\ \Delta_{sj},\ j = \overline{1,\ 3}),$$

and is itself a random variable. However, if we examine a volume V_n which includes n elementary volumes, then the probability approaches zero that, with an increase in n, the value of the physical characteristic ϕ_n of the volume V_n will deviate from a certain value taken as the characteristic of the material. Since Φ_n is represented in the form $\Phi_n = \sum_{i=1}^{n} \varphi_i/n$, then in accordance with the law of large numbers as stated by Hinchin:

$$\Phi_n \xrightarrow[n\to\infty]{P} M[\varphi_i(\xi)].$$

Thus, as the physical characteristic of the material, we will henceforth take the mathematical expectation of the characteristic for the elementary volume.

We determine the random vector of the parameters ξ in the space of elementary events Ω:

$$\Omega = \{\omega \mid \Delta_j = x_{sj},\quad l_{fj} = y_{ij},\quad l_{sj} = z_{pj},\quad \rho_j = c_{sj},\quad h_{sj} = f_{rj},$$
$$\Delta_{sj} = t_{mj},\quad j = \overline{1,\ 3};\quad k = \overline{1,\ K};\quad l = \overline{1,\ L};$$
$$p = \overline{1,\ P};\quad s = \overline{1,\ S};\quad r = \overline{1,\ R};\quad m = \overline{1,\ M}\},$$

Here, the mathematical expectation of the random vector function is determined naturally [11]:

$$M[\varphi(\xi)] = \sum_{\omega\in\Omega} \varphi(\xi(\omega))\, p(\omega).$$

The probability of an elementary event $p(\omega)$ is calculated through the probability characteristics of the structure-forming elements (fibers):

$$p(\omega) = \prod_{j=1}^{3} \{P(\rho_j = c_{sj})\, P(\Delta_j = x_{sj}/\rho_j = c_{sj})\, P(l_{fj} = y_{ij}/\Delta_j =$$
$$= x_{sj},\ \rho_j = c_{sj})\, P(l_{sj} = z_{pj}/l_{fj} = y_{ij},\ \Delta_j = x_{sj},$$
$$\rho_j = c_{sj})\, P(h_{sj} = f_{rj},\ \Delta_{sj} = t_{mj}/l_{sj} = z_{pj},$$
$$l_{fj} = y_{ij},\ \Delta_j = x_{sj},\ \rho_j = c_{sj})\}.$$

Using the above method, we can calculate any physical characteristic of a fiber composite if we have the mathematical model of this characteristic for an elementary volume.

Let us examine mathematical models of the physical characteristics of an elementary volume which go into the equations that describe radiative-conductive heat transfer.

1. Apparent density is determined as the ratio of the mass of fragments of the elementary volume to its value:

$$\rho_{e.v}(\xi) = \Big(\sum_{j=1}^{3} l_s \Delta_j^2 \rho_j\Big) \Big/ \prod_{j=1}^{3} l_j.$$

2. Volume heat capacity is equal to the ratio of the total heat capacity of the fragments of the elementary volume to its value:

$$c_{e.v}(\xi) = \left(\sum_{j=1}^{3} c_j l_{sj} \Delta_j^2\right) \Big/ \prod_{j=1}^{3} l_j.$$

3. Thermal conductivity. Following [12], we have adiabatic (parallel to the heat flow) and isothermal (perpendicular to the heat flow) surfaces subdivide the elementary volume into fragments and we determine the thermal resistances for these fragments:

$$R_{e.vi}(\xi) = \frac{1}{\lambda_{e.vi}} \frac{l_{e.vi}}{s_{e.vi}},$$

where $\lambda_{e.vi}$, $l_{e.vi}$, and $s_{e.vi}$ are thermal conductivity, height, and area of a fragment. Proceeding on the basis of a scheme in which the thermal resistances of the different fragments are connected, we calculate the thermal resistance of the elementary volume $R_{e.v}(\xi)$ and its thermal conductivity:

$$\lambda_{e.v}(\xi) = \frac{l_3}{R_{e.v}(\xi)\, l_1 l_2}.$$

For the adiabatic subdivision, the thermal resistance of the elementary volume (Fig. 2) is calculated from the formula:

$$\frac{1}{R_{e.v}} = \frac{1}{R_1 + R_6} + \frac{1}{R_2 + R_7} + \frac{1}{R_3 + R_5 + R_8} + \frac{1}{R_4}.$$

4. The absorption and scattering coefficients are determined as the ratio of absorption and scattering on elements of the fiber skeleton to the absorption and scattering in the elementary volume:

$$\alpha_{e.v}(\xi) = \left(\sum_{j=1}^{2} l_{sj} \Delta_j k_{absp\,j}\right) \Big/ \prod_{j=1}^{3} l_j,$$

$$\beta_{e.v}(\xi) = \left(\sum_{j=1}^{2} l_{sj} \Delta_j k_{sct\,j}\right) \Big/ \prod_{j=1}^{3} l_j.$$

The absorption and scattering efficiency factors k_{absp} and k_{sct} are calculated in accordance with the Mie theory, which has been used with success to calculate the optical characteristics of fiber insulation [5, 12]. Playing an important role here are various approximations of the Mie theory [8], which along with being accurate significantly facilitate the computing operation.

The refractive index is equal to the sum of the refractive indices of the fibers when they have been assigned weights equal to the volume fractions of fibers entering into the composition of the elementary volume:

$$n_{e.v}(\xi) = \left[\sum_{j=1}^{3} l_{sj} \Delta_j^2 n_j\right] \Big/ \prod_{j=1}^{3} l_j.$$

The nonlinearity of the heat-transfer model (1-7) and the large volume of computations necessary to determine the characteristics makes it mandatory to use numerical methods to calculate RCT in highly porous fiber composites. Mathematical models of the processes and the characteristics were realized in the form of application packages written in FORTRAN. One feature of the method being used here to calculate physical characteristics is the common approach to the computation, which makes it possible to develop an efficient algorithm for performing it. An analysis of the sensitivity of the models that was made using the software we developed confirmed the need to make allowance for features of the structure of the material when mathematically modeling RCT in HPC's [1].

Theoretical-Experimental Investigation. In the final stage of our study, we must answer the question of the adequacy of the proposed mathematical model with regard to its description of actual heat-transfer processes occurring in HPC's. As the object of investigation, we chose a fibrous quartz ceramic with the following parameters: porosity $\Pi = 0.95$; mean diameter $d_{av} = 2\mu m$; mean fiber length $l_{fav} \approx 400$ μm; anisotropy ≈ 2; ideal contact. The thermophysical and optical properties of the fibers and their distributions according to length and diameter were all known.

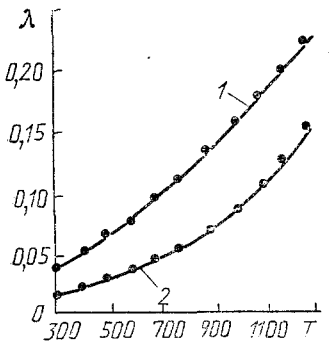

Fig. 3. Dependence of the effective thermal conductivity of a material based on SiO_2 fibers on temperature: points denote experimental results; curves show calculated results; 1) p = 760 mm Hg; 2) 0.1.

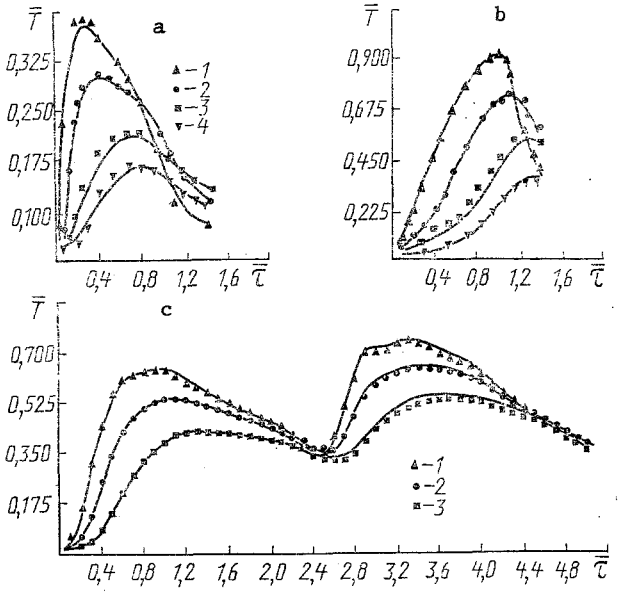

Fig. 4. Comparison of theoretical and experimental temperature during the unsteady heating of the fibrous material: a, b) first specimen; 1) $\bar{x}$ = 0.08; 2) 0.28; 3) 0.58; 4) 0.78; c) second specimen: 1) $\bar{x}$ = 0.13; 2) 0.28; 3) 0.5.

The effective thermal conductivity of the material was determined on a unit which employed the "hot-wire" method. Figure 3 shows the results of the calculation and experimental data on the effective thermal conductivity in the temperature range T = 300-1300 K and the range of relative pressures $p = 1\text{-}10^{-4}$. The good agreement between the theoretical and experimental data reinforced confidence in the model and allowed us to proceed to the study of nonsteady heating regimes. Here, the tests were conducted for two specimens in the temperature range T = 290-1400 K with an air pressure $P = 10^5$ N/m^2. The specimens, with a thickness ℓ = 0.06 m, were heated with a resistance heater. The experiments differed in the character and rate of heating and cooling and the temperatures reached on the front surface of the specimen. The temperature field in the material was monitored with platinum–platinum–rhodium thermocouples with a diameter d = 0.1 mm. The thermocouples were installed at different depths in the layer. The temperature measurements were estimated to have been accurate to within several degrees [5]. The readings of the thermocouples located on the front and rear sides of the specimens were used as first-order boundary conditions when we performed the computations. The thermocouples were installed at the relative depths $\bar{x}$ = 0, 0.08, 0.28, 0.58, 0.78, for the first specimen and $\bar{x}$ = 0; 0.13, 0.28, 0.5, for the second.

An analysis of the test results (Fig. 4) shows the good agreement between the theoretical and experimental data for all heating regimes. This confirms both the adequacy of the model "as a whole" and the adequacy of its constituent parts (such as the conductive part of the model in low-temperature tests).

Thus, we have provided the main components needed to solve thermal problems encountered in the development of high-porosity fiber composites: a model of the structure of the material; a model of the physical characteristics; a model of the heat-transfer processes; a model of the thermal loading conditions; testing equipment needed to simulate the thermal loading conditions that will be encountered in service.

NOTATION

c, volume heat capacity; λ, thermal conductivity; α, β, b, spectral absorption coefficient, spectral scattering coefficient, spectral attenuation coefficient; n, refractive index; ε, emissivity; T, temperature; Q_p, F, integral and spectral radiative heat flux; I_p, Planck function; ℓ, size of elementary volume, thickness of layer of fibrous material; Δ, transverse dimension of fiber in the model structure; ℓ_f, length of fiber; ℓ_s, length of element of the fiber skeleton in an elementary volume; P, pressure; M, symbol for mathematical expectation; ρ, density; R, thermal resistance.

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